

BIO 682
Nonparametric Statistics
Spring 2010

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<http://www4.nau.edu/shustercourses/BIO682/index.htm>

Lecture 7

***k*-Sample Tests**

1. Tests in which population is sampled *multiple times*.
2. Good example: Cochran's Q test.
 - a. A test for nominal scale data that tests for *changes over time*
 - b. Similar to McNemar's test, except that duration is not limited to two samples.

Cochran's Q Test

1. Faculty responses to New Plan at various times in over the last few months.
 - a. Possible to examine the effect of time on subjects.
 - b. Useful to have a control in most cases (not always possible).

Cochran's Q Test: Method

1. examine matrix with:
 - a. a = # of columns (sampling events)
 - b. b = # of rows (subjects)
 - c. Y = score for each individual at time a_i (0 or 1)

b_i	a_1	a_2	a_3	Σ
1	0	1	0	1
2	0	0	0	0
3	1	1	0	2
4	1	0	0	1
5	1	1	0	2
6	1	0	0	1
7	0	0	0	0
8	1	1	1	3
9	1	1	0	2
10	1	0	0	1
Σ	7	5	1	13
	49	25	1	75

Cochran's Q Test: Method

Calculate:

1. $\sum_{i=1}^a Y_i = 13 = A$
2. $\sum (\sum_{i=1}^a Y_i)^2 = 25 = B$
3. $\sum (\sum_{i=1}^b Y_i)^2 = 75 = C$

Cochran's Q Test: Method

$$Q = \frac{(a-1) [a(C) - (A)^2]}{a(A) - B}$$

$$= \frac{(3-1) [3(75) - 169]}{3(13) - 25}$$

$$= 8.$$

with $df = (a-1) = 2$, $(\chi^2 = 5.99)$, $P < .05$
 People change.

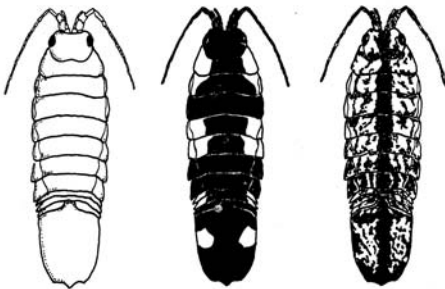
RxC Tests

1. Like a contingency table but with *more than* 2 rows or columns.
2. The classic method: RxC χ^2 test.
 - a. Method:
1. Marginal values calculated as with 2x2 test.
 2. Add up all χ^2 values for cells.
3. Has same problems with being cumbersome as 2x2 χ^2 test.

RxC G-tests

1. Has same advantages as before, same rules:
 - a. for $a > 5$ and $f_{i\text{-hat}} > 3$; G is *better* than χ^2
 - b. Use an exact test when:
 - a. $a > 5$ and $f_{i\text{-hat}} < 3$
 - b. $a < 5$ and $f_{i\text{-hat}} < 5$
2. Commonly used test to examine independence of multiple classes.

Example: *Idotea baltica*



uniformis

albufusca

maculata

Example: *Idotea baltica*

	M	J	J	A	
uniformis	254	185	93	55	587
albafusca	185	144	123	190	642
maculata	66	98	200	305	669
	505	427	416	550	1898
obs. <i>f</i> mac.	.13	.23	.48	.55	avg = .35

Example: *Idotea baltica*

1. RxC test allows you to test the hypothesis that the observed frequencies *don't change*.

2. Same method as 2x2:

$$[(\Sigma G\text{-cells}) - (\Sigma G\text{-rows}) - (\Sigma G\text{-columns}) + (G-N)]$$

a. with $df = (r-1)(c-1) = 6$.

3. Williams' correction is used for sample sizes <200.

a. Is a lot more complicated than before (see p. 745).

Williams' Correction: RxC

$$q = \frac{\sum_{i=1}^b \frac{1}{\sum_{j=1}^a f_{ij}} - 1}{6n(a-1)(b-1)}$$

But the shorter version provides a lower boundary (a conservative substitute),

$$q = 1 + [(a+1)(b+1)]/6n$$

RxC and Heterogeneity Tests

1. They are functionally analogous.
2. They both test whether the samples differ in their *observed frequencies*.
3. The difference is that heterogeneity tests are based on an *extrinsic hypothesis*.
4. RxC tests are based on marginal totals, therefore the hypothesis is *intrinsic* to data.

RxC: What is the Question?

1. Do the relative frequencies *change*?
2. A significant *G*-value tells you that differences **DO** exist among categories.
 - a. But, doesn't say much about *where* they are.
3. To answer this question, it is possible to collapse RxC into a series of 2x2 tests.

Collapsing Cells

1. It is necessary to pool adjacent rows and/or columns to reduce the number of comparisons.

	M	J	J	A	
uniformis	254	185	93	55	587
albafusca	185	144	123	190	642
maculata	66	98	200	305	669
	505	427	416	550	1898
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Measures of Association

1. The converse of tests of independence are tests of association.
 - a. If H_0 is rejected, inference can be that factors are associated.
 - b. Examples:
 1. Nonparametric correlations (Spearman's r ; Kendall's tau).
 2. Friedman's test
2. Three-way, multiple way contingency tables.

k-Way Tables

1. Multi-way contingency tables
 - a. Like ANOVA; they test the effect of multiple factors on observed values
 - b. However,
 1. ANOVA is concerned with *main effects*
 - a. If interactions are found, it is often difficult to identify their source.
2. Multi-way tables are specifically concerned with identifying source of interactions.

Ordinal Scale Tests

1. One sample cases- Runs test
 - a. There are many cases in which the order in which events occur is of interest.
 - b. Concern with independence, randomness.
 - c. Individuals choosing different sides of an experimental chamber.
 - d. Sequence in which different sexes defend territory.
2. Whenever it is possible to record the order in which events occur.

Coin Flips

1. Possible extremes in 20 tosses:
 - a. All heads or all tails.
 - b. 10 heads followed by 10 tails
- c. T H T H T H T H T H T H T H T H T H T H T H T H
2. More likely, there is some intermediate pattern:

H H T T T H T T T H H H T H T T T H
3. In each case it is possible to count the number of "runs" that occur (r).

Counting Runs

1. The first two cases have *fewer* runs than expected by chance ($r=1$ and 2)
2. The third has *more* runs than expected by chance ($r=20$)
3. the fourth has $r=9$.
4. The number of runs (r) will depend on:
 - a. m - # of events of one type
 - b. n - # of events of the other type
- c. Since these variables count all events,

$$N = r = n + m.$$

TABLE G:
Critical values of r in the runs test*

Given in the table are various critical values of r for values of m and n less than or equal to 20. For the non-sample size, use any observed value of r which is less than or equal to the smaller value, or is greater than or equal to the larger value in a pair in significance at the $\alpha = .05$ level.

$n \backslash m$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	1	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4	1	2	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
5	1	2	3	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
6	1	2	3	4	5	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
7	1	2	3	4	5	6	7	7	7	7	7	7	7	7	7	7	7	7	7	7
8	1	2	3	4	5	6	7	8	8	8	8	8	8	8	8	8	8	8	8	8
9	1	2	3	4	5	6	7	8	9	9	9	9	9	9	9	9	9	9	9	9
10	1	2	3	4	5	6	7	8	9	10	10	10	10	10	10	10	10	10	10	10
11	1	2	3	4	5	6	7	8	9	10	11	11	11	11	11	11	11	11	11	11
12	1	2	3	4	5	6	7	8	9	10	11	12	12	12	12	12	12	12	12	12
13	1	2	3	4	5	6	7	8	9	10	11	12	13	13	13	13	13	13	13	13
14	1	2	3	4	5	6	7	8	9	10	11	12	13	14	14	14	14	14	14	14
15	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	15	15	15	15	15
16	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	16	16	16	16
17	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	17	17	17
18	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	18	18
19	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	19
20	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

* Adapted from David and Forstner, C. (1975). Tables for testing randomness of grouping in a sequence of alternative events. *Journal of Mathematical Statistics*, 18, 91-96, with the kind permission of the authors and publisher.

Small Samples

1. Where m and $n < 20$
 - a. Use Table G (S&C)
 - b. Provides the values for m and n
- c. Also, boundaries of values for r that could occur 95% of the time.
 1. Thus, provides a *2-tailed test*.
2. If a 1-tailed test: H_0 is rejected is at $\alpha = .025$

Large Samples

1. When m and $n > 20$:
 1. The value of z is based on a normal distribution.

$$z = \frac{r + h - \mu_r}{\sigma_r}$$

where r = # of runs

Where,

- a. $\mu_r = (2mn/N) + 1$
- b. $\sigma_r = \sqrt{\{[2mn(2mn - N)] / [N^2(n-1)]\}}$
- c. $h = .5$ if $r < [(2mn/N)+1]$ and $-.5$ if $r > [(2mn/N)+1]$.

Two Sample Cases

1. The Sign Test
 - a. One of the simplest tests using ordinal data.
 - b. Is used like a binomial test to determine the *order* of two samples.

Example: Sign Test

1. The number of warning cries delivered against intruders by male and female pairs of trogons.